Playsheet 2
Logic and Sets

MATH 130
20 points

Directions: Groups should consist of three or four people. Work together on each problem; do not delegate different problems to different people. Submit one **neatly written** write-up per group on the due date, and make sure all group members' names appear on the submission. Use complete sentences and explain your reasoning.

We need a few basic tools and a common language to help us communicate. We'll start with some definitions:

- **Set:** A **set** is a collection of objects that is described in such a way that any given object is either a part of the collection or it is not. The objects in the set are its **elements** or **members**. We usually name sets using capital letters; e.g., the set S or the set A. We use { and } to indicate the elements of a set: $S = \{2, 7, \text{carrot cake}\}$ says that a set called S contains the elements 2, 7, and carrot cake. The order we list the elements doesn't matter, and duplicates don't count: $\{2, 2, 2\} = \{2\}$.

- **Subset:** A **subset** of a set A is another set B whose elements are all elements of A. The set $\{2, 7\}$ is a subset of the set S above, but {carrot cake, chocolate cake} is not.

- **Union:** The **union** of two sets A and B is the set $A \cup B$ made up of all elements that are in A or B or both. $\{2, 7, \text{carrot cake}\} \cup \{\text{carrot cake, chocolate cake}\}$ is the set $\{2, 7, \text{carrot cake, chocolate cake}\}$.

- **Intersection:** The **intersection** of two sets A and B is the set $A \cap B$ made up of those elements that belong to both A and B. $\{2, 7, \text{carrot cake}\} \cap \{\text{carrot cake, chocolate cake}\}$ is the set {carrot cake}.

- **Statement:** A **statement** is a declarative sentence that is definitively either true or false, but not both. Note that this does **not** mean that we know which is the case, only that one is. We usually assign lower case letters as names of statements; e.g., the statement p or the statement q. For example: let p be the statement, "Colin has three cats today," and let q be the statement, "Colin has no dogs."

- **Conditional statement:** A **conditional statement** is a statement of the form "if (statement 1), then (statement 2)." A conditional statement is true if either (1) statement 1 is false, or (2) statement 1 is true and statement 2 is true. It's like a promise: if you bring me a cookie, I will give you half of it. If you bring me a cookie and I give you half of it, I have lived up to my promise. If you don't bring me a cookie (i.e., statement 1 is false), I don't have to give you half of a cookie, but I have still held up my end.

- **Conjunction:** The **conjunction** of two statements p and q is the statement "p and q" that is true precisely when p and q are both true. I do have three cats today, and I have no dogs, so the conjunction "Colin has three cats and Colin has no dogs" is true.

- **Disjunction:** The **disjunction** of two statements p and q is the statement "p or q" that is true precisely when p is true or q is true or both are true. I do have three cats today, and I have no dogs, so the disjunction "Colin has three cats or Colin has no dogs" is true. So is the disjunction, "Colin has three cats or Colin has 4712 dogs."

1. We have need of an "empty" set from time to time: a set with no elements. Design a useful notation to indicate such a set.

2. List all subsets (including the empty set) of the set $A = \{1, 2\}$. How many are there?

3. List all subsets (including the empty set) of the set $B = \{1, 2, 3\}$. How many are there?

4. Make an educated guess about how many subsets of the set $C = \{1, 2, 3, 4\}$ there are without actually listing them out. How could you convince your group that you are right (again without listing them out)?

5. Which sentences below are statements? Explain your decision in each case. If a sentence is a statement, determine whether it is true or false or you can't determine which.

   (a) Colin Starr teaches math.
   (b) All dogs have two heads.
   (c) It is hot outside.
   (d) This sentence is false.
   (e) There is life in other solar systems.

6. Write a conditional statement.

7. The converse of a conditional statement "if p then q" is the statement "if q then p." What is the converse of the conditional statement you wrote in the previous problem? Do your statement and its converse mean the same thing?

8. We are primarily interested in **arguments**. An **argument** is a sequence of statements (premises) that are logically connected to each other and lead to a conclusion: all mathematicians are hilarious; Colin is a mathematician; therefore, Colin is hilarious.

   We will be thinking about how to determine whether an argument is **valid**: do the premises lead to the conclusion? Note the important distinction here: we are not determining whether the conclusion is **true** – only whether it follows from the premises.

   Determine which arguments are valid.

   (a) All mathematicians are hilarious; Colin is a mathematician; therefore, Colin is hilarious.
   (b) All flurbims are snepzy; Twazzle is a flurbim; therefore, Twazzle is snepzy.
   (c) Most homeless people are male. Most [2023 update: a plurality of] homeless people are white. Therefore, white males are most at risk of homelessness. [1]

9. Note the parallels between the definitions for sets and the definitions for logic. Which terms correspond to each other, and how?

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[1] https://endhomelessness.org/homelessness-in-america/homelessness-statistics/state-of-homelessness/, https://www.statista.com/statistics/555855/number-of-homeless-people-in-the-us-by-race/